π-Josephson Junction and Spontaneous Superflow in Rings from Ultracold Fermionic Atomic Gases

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The BCS-like pairing in ultracold fermionic atomic ($UCFAG$) gases is studied in the model of "isotopic-spin" pairing proposed in 1991 [1]. This model assumes a mismatch (δ) in chemical potentials of pairing fermionic atoms. It is shown that a π-Josephson junction can be realized in $UCFAG$ systems, where the left and right banks S are the $UCFAG$ superfluids. The weak link M consists from the normal $UCFAG$ with the finite mismatch δ. If the π-junction is a part of a closed ring the superfluid mass-current flows spontaneously in the ring, i.e., the time-reversal symmetry is broken spontaneously. This is realized if the radius of the ring R is larger than the critical one R_c. All these effects exist also in the case when $\delta \gg \Delta$, where Δ is the superfluid gap, but with the reduced thickness of the weak link.

It is also discussed, that if junctions SM_1M_2S and trilayers M_1SM_2 from $UCFAG$ are realizable this renders a possibility for a novel electronics - *hypertronics*.

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I. INTRODUCTION

Physics of ultracold atoms is fascinating in many respects. The ways how these systems can be manipulated in magnetic and optical traps open possibilities to an enormous number of physical effects which can be realized and studied in these systems [2], [3]. Thanks to a large variety of experimental techniques one can manipulate atomic gases by magnetic and electromagnetic fields giving rise to realization of various macroscopic quantum effects. First of all, a number of effects known in standard superfluids and superconductors were already realized in ultracold atomic gases [2], [3]. In that sense a number of already developed theoretical methods and ideas in solid state physics were used in physics of ultracold atoms. On the other hand this fascinating field gives us new possibilities in studying many aspects of the Bose-Einstein condensation (BEC) and of the superfluidity of Fermi condensates (BCS) which are difficult to realize in solid state physics. For instance in an ultracold gas of bosonic atoms, such as ^{87}Rb or ^{7}Li, one can study Bose-Einstein condensation(BEC). If instead we deal with a gas of *fermionic atoms*, such as ^{40}K and ^{6}Li, one can study not only fermionic superfluidity (BCS) but also the transition BEC-BCS by directly controlling the interaction (scattering length) between atoms [3]. Then by tuning magnetic field in magnetic traps and/or electromagnetic field in optical lattices one can vary the atomic scattering length a_F in a broad range (especially for energies just near the Feshbach resonance), that even BCS-like pairing of ultracold atoms can be realized with $a_F < 0$. It seems that this possibility is already realized with ultracold fermionic alkali gases ^{40}K and ^{6}Li [4], [5]. The cooling of a magnetically trapped spin-polarized ^{6}Li Fermi gas up to 7×10^7 atoms at $T < 0.5\ T_F$ and up to 3×10^7 atoms at $T \approx 0.05\ T_F$ is already realized. The strength of the atomic interaction was controlled by applying a magnetic field and tuning to Feshbach resonances, which occur when the total energy of interacting particles in open channels is the same as the energy of a bound molecular state in closed channels.

For such a two-level fermionic system it was proposed by the present author and his collaborator in the paper "Inhomogeneous Superconducting Phase in Absence of Paramagnetic Effect" a BCS-like model for the *"isotopic spin" pairing* of two (or more) species [1] - the ISP model. The basic ingredient of the ISP model is that two species (atoms, electrons, quarks, nucleons, etc.) may have either different kinetic energies, or different energy levels (or chemical potentials), i.e there is a mismatch of energy levels of fermionic atoms which participate in pairing. It turns out that this model is an adequate theoretical framework for a number of experimental situations which deal with ultracold fermionic alkali gases, such as for instance ^{40}K and ^{6}Li . The present experimental techniques allow realizations of systems with two (or more) hyperfine levels. They can be furthermore manipulated in order to maintain various kinds of atomic pairing from BCS-like to strong coupling case as well as to realize the mismatch between the paired "isotopic" atomic levels (chemical potential). For further application of this model see below.

In the following we apply this model in studying nonuniform superfluidity in ultracold fermionic atomic gases ($UCFAG$) with oscillating order parameter $\Delta(\mathbf{r})$ - which is due to the mismatch effect of two hyperfine states of fermionic atoms. This effect is analogous to the Larkin-Ovchinikov-Fulde-Ferrell ($LOFF$ or $FFLO$) phase in superconductors [6], [7]. Based on this effect we propose a novel Josephson junction (SMS) where the left and right superfluid (S) gases have uniform order parameters $\Delta_{L,R} = const$, while the weak link (M) with the mismatch effect is in the normal state. This gives rise to an oscillating superfluid amplitude inside the weak link

M and as a result the so-called π-junction can be realized. We show that if such a junction is a part of the closed ring then *spontaneous and dissipationless superfluid current* can flow through the ring depending on the size of the ring. In that case there is *spontaneous breaking of the time-reversal* in the system. Finally, we discuss possible realizations and generalizations of this novel effect in ultracold fermionic gases.

II. MODEL OF ISOTOPIC-SPIN PAIRING AND ULTRACOLD FERMIONIC GASES

The model with the "isotope-spin" pairing (ISP) [1] is a generalization of the BCS pairing mechanism to systems with internal degrees of freedom such as for instance, *nuclear matter* - with isospin numbers, *quark matter* - with the color and flavor quantum numbers, *ultracold fermionic gases* - with quantum numbers of the hyperfine atomic states, *layered* and *multiband superconductors* - with quantum numbers enumerating layers and bands. The pairing constituents (electrons, nucleons, quarks, neutral atoms, etc.) can be either charged or not and in those cases we deal either with real superconductivity or with superfluidity of matter. All these systems posses a natural mismatch in energies (or masses) of constituents participating in pairing. This is an important property since in the case when the mismatch parameter δ is of the order of the bare superconducting gap Δ_0 a nonuniform superfluidity (superconductivity) is realized - an analogue of the $LOFF$ state in metallic superconductors placed in the Zeeman field. The Hamiltonian of the ISP model is given by [1]

$$\hat{H} = \hat{H}_0 + \hat{H}_{BCS} \tag{1}$$

$$\hat{H}_0 = \sum_{a=1,\sigma}^{n} \int d^d x \hat{\psi}^{\dagger}_{a\sigma}(\mathbf{x}) \varepsilon_{a\sigma}(\hat{\mathbf{p}}) \hat{\psi}_{a\sigma}(\mathbf{x}) \tag{2}$$

$$\hat{H}_{BCS} = -\sum_{a,b} g_{ab} \int d^d x \hat{\psi}^{\dagger}_{a\uparrow}(\mathbf{x}) \hat{\psi}^{\dagger}_{b\downarrow}(\mathbf{x}) \hat{\psi}_{b\downarrow}(\mathbf{x}) \hat{\psi}_{a\uparrow}(\mathbf{x}) \tag{3}$$

The ISP order parameter $\Delta_{ab}(\mathbf{x})$

$$\Delta_{ab}(\mathbf{x}) = g_{ab} F_{ab}(\mathbf{x}) = -g_{ab} \langle \hat{\psi}_{a\uparrow}(\mathbf{x}) \hat{\psi}_{b\downarrow}(\mathbf{x}) \rangle. \tag{4}$$

In the following we shall study the BCS-like pairing and Josephson effect in ultracold fermionic alkaline gases ($UCFAG$) in which case $a, b = 1, 2$ and $\Delta_{ab} \neq 0$ for $a \neq b$ only - see the explanation below, while the electronic spin states $\uparrow$ and $\downarrow$ do not enter (they are accounted for via a, b). For simplicity we omit indices a, b in $\Delta_{ab}(\mathbf{x})$, i.e. $\Delta_{ab}(\mathbf{x}) \equiv \Delta(\mathbf{x})$. In the case of two ($a = 1, 2$) "isotopic" bands with the mismatch in their energies (or chemical potentials) - the *"isotopic" band splitting* (δ) - the quasiparticle spectrum is given by

$$\varepsilon_{a=1,2}(\hat{\mathbf{p}}) = \varepsilon(\hat{\mathbf{p}}) \mp \delta. \tag{5}$$

Note, that the splitting parameter δ can in principle depend on momenta too, but the results obtained below are qualitatively valid also in this case.

It is interesting to mention that the above model and some physical results [1] were frequently used later on in studying diverse physical problems such as: (1) *ultracold fermionic atomic gases* - where $a, b = 1, 2, ..n$ enumerate hyperfine atomic states of alkali atoms; (2) in *quantum chromodynamics* - where the problem of pairing in the quark matter, i.e., the *color superconductivity* (CS), was studied [8]. In the latter case the CS pairing of the colored quarks is mathematically more complicated than the standard BCS case, since the former system is characterized by the matrix coupling constant g_{ab} in the internal degrees of freedom, where $a, b = (c, f, \sigma)$ are quantum numbers of quarks: $c = 1, 2, 3$ - *color*; $f = 1, 2...n$ - *flavor* (up, down, strange,...); $\sigma = \uparrow, \downarrow$ - *spin*. At the same time the Hamiltonian $\hat{H}_{CS}$ (for instance the Nambu-Lasinio Hamiltonian), which is an analogue of the BCS one in the CS pairing, must be invariant under the symmetry group of strong interactions $SU(3)_c \otimes SU(3)_{f,L} \otimes SU(3)_{f,R} \otimes U(1)$ - see [8].

Let us mention some application of this model in *the solid state physics* such as *bi-layered superconductors* in the presence of the Zeeman effect ($h = \mu_B H$)

$$\hat{H}_Z = h \sum_{a=1}^{2} \int d^d x [\hat{\psi}^{\dagger}_{a,\uparrow}(\mathbf{x}) \hat{\psi}_{a,\uparrow} - \hat{\psi}^{\dagger}_{a,\downarrow}(\mathbf{x}) \hat{\psi}_{a,\downarrow}(\mathbf{x})]. \tag{6}$$

It turns out that the paramagnetic field h can compensate the "isotopic" mismatch of the chemical potentials ($\delta \neq 0$) giving rise to some novel effects [1]. For instance, in case when $\delta > \Delta_0$ superconductivity is destroyed by the isotopic-splitting δ, while the Zeeman term can compensate this mismatch and thus inducing superconductivity for $\delta - h < \Delta_0$ - the *reentrant superconductivity* [1]. Recently, similar effects were found to exist in layered superconductors placed in the Zeeman field h [9]. In that case the bi-layer superconductivity was studied in the model (similar to that in Eq.(2-3)) with the *intralayer* pairing Δ only. In that case the interplane hoping (t) in the bi-layer system plays the role of the "isotopic" splitting parameter, i.e. $\delta = t$, and $\varepsilon_{a=1,2}(\hat{\mathbf{p}})$ are the bonding (B) and antibonding (A) bands, respectively. It turns out that for $h < \Delta$ the bi-layer superconductivity is realized, i.e, $\Delta \neq 0$, while for $h > \Delta$ it is destroyed, i.e., $\Delta = 0$. However, for $h \approx t (\gg \Delta_0)$ superconductivity appears again (reentrant superconductivity) on the expense of the π-phase difference between the order parameters on the bilayer (in the up- and down-plane), i.e. $\Delta_{up} = -\Delta_{down}$. In the language of the ISP model there is a pairing between the antibonding and bonding levels only, i.e., $\Delta_{AB} \neq 0$ while $\Delta_{AA} = \Delta_{BB} = 0$. This result

is of a potential interest in high-temperature superconductors, especially in bi-layer BISCO.

One expects that similar effects can be realized also in the *quark matter* with the color superconductivity (CS) with two flavor quarks, such as pulsars-magnetars, where the CS gap is $\Delta \sim 10\ MeV$ which is of the order of mass difference $\delta M \sim 5\ MeV$ between up and down quark. Having in mind that the magnetic field in magnetars is huge, $B > 10^{15}G$, then the above compensating effect (by magnetic field and Zeeman effect) might be operative there.

The ISP model renders a number of interesting effects and some of them were discussed in [1]. In the following we study systems with two levels $a = 1, 2$ only and apply it to the ultracold fermionic gases. It is assumed pairing between two species $\Delta_{ab} \neq 0$ for $a \neq b$, only. This two-state model is realistic approximation since in diluted gases, for instance in ^6Li, the interaction of atoms in different hyperfine states is several order of magnitude larger than in the same states [13]. Recently, the fermionic superfluidity was realized recently by several groups [14]. However, the definitive experimental prove of the fermionic superfluidity was given recently in the remarkable experiment of the Ketterle's group [15]. They cooled ^6Li in magnetic trap below the Fermi degeneracy. This Fermi cloud, consisting of approximately 10^6 atoms with two lowest hyperfine states $\mid 1\rangle$ and $\mid 2\rangle$, was loaded into an optical dipole trap. Between these states there is a Feshbach resonance at $B_0 = 875\ G$. The BEC-BCS transition occurs in the region between 780 G and 925 G, where for $B > B_0$ the system is in the BCS side. Since the (attractive) coupling between atoms in states $\mid 1\rangle$ and $\mid 2\rangle$ at low T is strong in the s-channel, this means that the spatial part of the two-body wave function $\Psi(a_1, a_2)$ is symmetric under the exchange of atoms, i.e. $\Psi(a_1, a_2) = \Psi(a_2, a_1)$. As the result the spin part of the wave function is antisymmetric, i.e.,

$$\mid \{1, 2\}\rangle = \frac{1}{\sqrt{2}}[\mid 1\rangle \mid 2\rangle - \mid 1\rangle \mid 2\rangle]. \tag{7}$$

The pairing is strong between the states $\mid 1\rangle$ and $\mid 2\rangle$, i.e., $\Delta_{12} \neq 0$ while $\Delta_{11} = \Delta_{22} = 0$. The crucial observation in [15] was the creation of vortices in the rotating systems above some rotation frequency, what can be considered as a definite prove for fermionic superfluidity.

Since the ISP model strongly reminds on metallic superconductors with the Zeeman (paramagnetic) effect, this analogy might be useful in studying ultracold fermionic gases. *First*, in the case when the "isotope" splitting is larger than the critical value $\delta_{c1} = 0.71\Delta^0_{ab}$, then in the region $\delta_{c1} < \delta < \delta_{c2}$ (at $T = 0\ K$) a *nonmagnetic analogue* of the $LOFF$ phase is realized with the nonuniform order parameter $\Delta(\mathbf{x}) \sim \cos \mathbf{Q}\mathbf{x}$. For $\delta = \delta_{c1}$ the modulation vector $\mathbf{Q}$ is given by $Q_{c1} = 2.4(\delta_{c1}/V_F)$ where V_F is the Fermi velocity. It is known that the structure of the order parameter may contain more wave vectors $\mathbf{Q}_i$ thus making the so called crystalline structure more favorable [6] - this (still unsolved) problem is not the subject of this paper. Concerning the critical value δ_{c2} it depends on dimensionality of the system - in 3D one has $\delta_{c2} = 0.755\Delta^0_{ab}$, in 2D $\delta_{c2} = \Delta^0_{ab}$ while for 1D $\delta_{c2} \to \infty$. Note, that for $\delta < \delta_{c1}$ (at $T = 0$ K) the system is in uniform superfluid (superconducting) state, i.e. $\Delta_{ab} = const$. The transition at δ_{c2} depends on the type of the nonuniform solution - for instance for the one plane solution we use here (only for simplicity) the transition at ($T = 0$, δ_{c2}) is second order. The phase diagram in the plane (T, δ) of the "isotopic" $LOFF$ phase is shown in $Fig.1$.

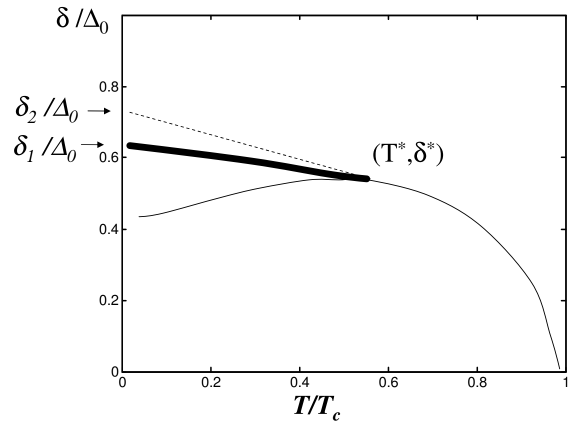

FIG. 1: The phase diagram (δ, T) of the 3D superfluid ultracold fermionic gas. At T^*, δ^* there is second order phase transition. The bold line is the first order line between normal state and uniform superconductivity. The region between the upper-dotted and the lower bold line is the nonuniform LOFF state.

Second, it turns out that the $LOFF$ state appears at temperatures $T < T^*$ and in the phase diagram there is a tricritical Lifshitz point ($T^* = 0.56T_c$, $\delta^* = 1.32T_c$). Near this point one can perform the Ginzburg-Landau (GL) expansion analogously to that obtained in [10], [1]. In order to prove the existence of the π- junction in the superfluid $UCFAG$ we shall study the nonuniform superfluidity near the point (T^*, δ^*) where it is straightforward to derive the GL expansion by using Eqs.(3-4). As the result the (dimensionless) density of the free-energy functional $\tilde{F}\{\psi(\mathbf{x})\} \equiv F/E_c$ (where $E_c = N(0)\Delta^2_0/2$ is the condensation energy and $\psi(\mathbf{x}) = \Delta(\mathbf{x})/\Delta_0$) is given by [10], [1]

$$\tilde{F}\{\psi(\mathbf{x})\} = \tau \mid \psi \mid^2 - \frac{\gamma}{2} \mid \psi \mid^4 + \frac{\nu}{3} \mid \psi \mid^6$$

$$- \alpha\xi_0^2 \mid \nabla\psi \mid^2 + \beta\xi_0^4 \mid \nabla^2\psi \mid^2 + \frac{\mu}{2}\xi_0^2 \mid \psi \mid^2 \mid \nabla\psi \mid^2 + ... \tag{8}$$

Here, $\tau = [T - T_c(\delta)]/T^*$, $\gamma = \tilde{\gamma}(\frac{\delta-\delta^*}{\delta^*} - \gamma'\tau)$, $\alpha = \tilde{\alpha}(\frac{\delta-\delta^*}{\delta^*} - \alpha'\tau)$, where $T_c(\delta)$ is the transition temperature from the normal (N) to the uniform BCS state but in the absence of the $LOFF$ state. $N(0)$ and ξ_0 are the density of states and superconducting (superfluid) coherence length (at $T = 0$), respectively. The parameters $\tilde{\alpha}$, α', $\tilde{\gamma}$, γ', ν and μ are of the order of one and their precise values can be found in [11]. Since the term in front of $\mid \nabla\psi \mid^2$is negative it leads to the nonuniform $LOFF$ state near the tricritical point (T^*; δ^*). The minimization of $F\{\psi(\mathbf{x})\}$ gives the transition temperature $T_{c,q}(\delta)$ from the normal to the LOFF state (the dotted line in $Fig.$1) and the wave vector Q

$$\tau(T_{c,q}) = \frac{T_{c,q}(\delta) - T^*}{T^*} = \frac{\tilde{\alpha}^2}{4\beta}(\frac{\delta-\delta^*}{\delta^*})^2 \qquad (9)$$

$$Q^2 = \xi_0^{-2}\frac{\tilde{\alpha}}{2\beta}\frac{\delta-\delta^*}{\delta^*}. \qquad (10)$$

In the $LOFF$ state the mass-current density ($\mathbf{j}^{(m)} \equiv \mathbf{j}/(-2e)$ and $\tilde{\mathbf{j}}^{(m)} = \mathbf{j}^{(m)}/E_c$) has much reacher form then the standard G-L expression

$$\tilde{\mathbf{j}}^{(m)} = i[-\alpha\xi_0^2 + \frac{\mu}{2}\xi_0^2 \mid \psi \mid^2]\psi^*\nabla\psi$$

$$+ \beta\xi_0^4 i\{\nabla[\psi\nabla^2\psi^*] - 2\nabla\psi\nabla^2\psi^*\} + c.c. \qquad (11)$$

In the case of charged superconductors the orbital effect of magnetic field can be accounted for by replacing $\nabla \to \nabla + (2ie/c)\mathbf{A}$. In the case of $UCFAG$, such as ^{40}K and ^6Li, the current $\mathbf{j}^{(m)}$ is the *mass-current*.

III. JOSEPHSON EFFECT IN ULTRACOLD FERMIONIC GASES

We shall demonstrate below that in principle it is possible to have a *π-Josephson contact (weak link)* based on cold fermionic gases which is analogous to the so called SFS contacts (weak links) in the solid state physics, where in the ferromagnetic weak link (F) electrons with different spin projections have different energies (Zeeman-mismatch). In this case the superconducting amplitude $F_{\uparrow\downarrow}$ is induced in F by the proximity effect. This amplitude not only decays (like in the case of normal metal SNS contacts) but also oscillates between two superconducting banks, i.e. $F_{\uparrow\downarrow}(x) \sim e^{-Q_1 x}\cos Q_2 x$ - see [16], [12]. The oscillation in the weak link, especially in the case $\delta \gg \Delta$, is in fact a reminiscence of the $LOFF$ state as it was shown in [10] by using Eqs. (8-11).

The Josephson weak-link with $UCFAG$ can be in principle realized by using various optical and magnetic trap techniques. If the $UCFAG$ with only two hyperfine levels are trapped their pairing interaction can be described in some parameter range by Eqs.(3-4). We further assume that the left and right part of the weak-link are similar and that the mismatch of the chemical potentials in them is small, $\delta \ll \Delta_{0,L} = \Delta_{0,R}$. In that case the left ($x < -L$) and right ($x > L$) BCS superfluid-condensates are uniform with the critical temperature T_s and the order parameters $\mid \Delta \mid e^{i\varphi_L}$ and $\mid \Delta \mid e^{i\varphi_R}$, respectively. Furthermore we assume that the weak-link M ($-L < x < L$) with the width $2L$ between the left and right banks is made of $UCFAG$ with the mismatch $\delta \neq 0$ but which is in the normal state. In the following we call such a weak-link SMS, where S means the BCS superfluid and M is the weak link with the mismatch parameter δ and at temperatures where the system is a *normal Fermi gas*. This means that in M the order parameter $\Delta_M = 0$ but the anomalous Green's function $F_{ab,M}(x) \neq 0$ due to the proximity effect [12]. At the end we shall argue that the Josephson effect can exist even in the case $\delta \gg \Delta$, i.e. far away of the $LOFF$ state. However, in this case one should use rather sophisticated microscopic many-body techniques - for instance the Eilenberger quasiclassical equations. In order to prove the existence of the π-Josephson junction we shall study the problem when T and δ are just near the tricritical point ($T^* = 0.55T_c$; $\delta^* = 1.32T_c$) - see $Fig.$1. In such a case the weak link M, which is in the normal state but very near to the LOFF state, is characterized by the induced order parameter ψ_M which is small near the point (T^*; δ^*) and the GL equation can be linearized. Since $\psi_M(x)$ changes along the x-axis which is perpendicular to the surface of the weak link (M) - see $Fig.$2, the GL equation in the weak link M reads

$$\tau\psi_M - \alpha\xi_0^2\frac{\partial^2\psi_M}{\partial x^2} + \beta\xi_0^4\frac{\partial^4\psi_M}{\partial x^4} = 0. \qquad (12)$$

In order to simplify the analysis we assume that $\tau \gg \alpha^2/\beta$ and the solution $\psi_M \sim e^{Q_M x}$ oscillates with $Q_M = \xi_0^{-1}(1 \pm i)(\tau/4\beta)^{1/4}$. The general solution in the weak link is

$$\psi_M(x) = \sum_{p=1,-1}(A_p e^{pQ_M x} + B_p e^{pQ_M^* x}) \qquad (13)$$

where A_p and B_p can be obtained from the boundary conditions at $x = -L$ and $x = L$. Since at present there is no microscopic theory for the Josephson effect in $UCFAG$ we use the experience from the physics of the SFS weak links. There various boundary conditions do not destroy the oscillations of $\psi(x)$ in the weak link, which is important property for the realization of the π-contact. Therefore, we assume that $\psi_M(x)$ and $\partial\psi_M/\partial x$ are continuous on boundaries at $-L$ and L.

Since we study the problem near the tricritcal point (T^*, δ^*) then the standard term ($\sim \psi_M\nabla\psi_M^*$) in the current is small and the linearized expression for the current $\tilde{j}_x^{(m)}$ is given by

$$\tilde{j}_x^{(m)} \approx i\beta\xi_0^4[\psi_M\frac{\partial^3\psi_M^*}{\partial x^3} - \frac{\partial\psi_M}{\partial x}\frac{\partial^2\psi_M^*}{\partial x^2} + c.c]. \qquad (14)$$

Due to the particle current conservation the Josephson current trough the weak link M can be calculated at the

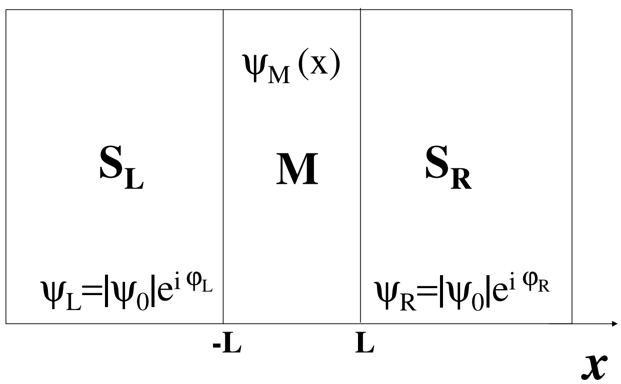

FIG. 2: The Josephson junction with uniform superfluidity in banks $S_{L,R}$ with $\mid \psi_{L,R} \mid = const$ and $\delta < \delta^*$. In the weak link M with the thickness $2L$, which is in the normal state (above the doted line in $Fig.$1), one has $\delta \neq 0$ and Ψ_M oscillates and decays. The Josephson current flows along the x-axis.

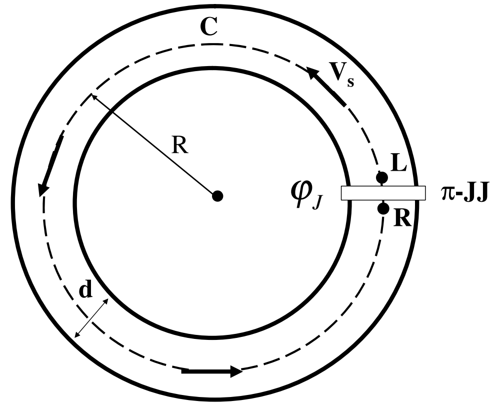

FIG. 3: The ring with the π-Josephson junction. ϕ_J is the phase on the contact. The closed contour is C and for $d << R$ the superfluid velocity V_s is uniform over the cross-section of the ring. For the radius $R > R_c$ a spontaneous current flow with the velocity V_s - the spontaneous breaking time-reversal symmetry.

midpoint $x = 0$, i.e., $\tilde{j}^{(m)}(\varphi) = \tilde{j}_x(x=0)$ is given by

$$\tilde{j}^{(m)}(\varphi) = \tilde{j}_c \sin\varphi, \tag{15}$$

where

$$\tilde{j}_c = 4 \mid Q_M \mid^3 \beta\xi_0^4 \mid \psi_0 \mid^2 e^{-\sqrt{2}|Q_M|L}$$

$$\times \sin(\sqrt{2} \mid Q_M \mid L - \frac{\pi}{4}). \tag{16}$$

Here, $\varphi = \varphi_L - \varphi_R$ is the phase difference on th junction. From Eq. (16) one concludes that $\tilde{j}_c$ can reach negative values whenever $\sin(\sqrt{2} \mid Q_M \mid L - \frac{\pi}{4}) < 0$. This can be achieved by changing either T, δ or the thickness L of the weak link M. In such a way one can realize a π-*junction* - the contact with $\tilde{j}_c < 0$. To remaind the reader, the π-contact is characterized by the phase difference $\varphi = \pi$ in the ground state, while in the standard Josephson contacts $\varphi = 0$ minimizes the energy of the contact. In the next Section we are going to show, that if the π-junction is placed in a ring with the large radius R there is a *spontaneous superfluid flow* through the ring, i.e. the *time-reversal symmetry is broken spontaneously.*

IV. π-JUNCTION AND SPONTANEOUS SUPERFLUID FLOW

Let us consider a loop made of an ultracold fermionic BCS superfluid with the π-contact placed in a ring of radius R as shown in $Fig.$3.

If the mass-current flows through the Josephson junction and the ring the total energy (per cross-section S of the ring) of the system $W(= W_K + W_J)$ is due to the kinetic energy $W_K = E_K/S$ of the *circulating superfluid current* and the energy $W_J = E_J/S$ of the Josephson contact. In the ring with a small cross-section ($d \ll R$) the superfluid velocity $V_s(r)$ is practically constant over the cross-section of the ring ($V_s(r) \approx V_s(R) \equiv V_s$) and the total energy is given by the simple expression

$$W = \frac{n_s(2m_a)V_s^2}{2}(2\pi R) - \frac{\hbar \mid j_c^{(m)} \mid}{2} cos(\varphi_J + \varphi_{int}). \tag{17}$$

Here, n_s is the density of superfluid pairs, m_a is the mass of the fermionic atom, and $\varphi_J = \varphi_L - \varphi_R$ is the phase difference on the contact. The intrinsic phase of the junction is $\varphi_{int} = 0$ for $j_c^{(m)} > 0$ and $\varphi_{int} = \pi$ for $j_c^{(m)} < 0$.

The relation between the superfluid velocity $\mathbf{V}_s = (\hbar/2m_a)\nabla\varphi$ and the phase φ_J can be obtained from the *quantization condition* of the superfluid phase φ along the closed path C in the ring - see $Fig.$3

$$2\pi n = \oint_C \delta\varphi = \int_{C'} \nabla\varphi d\mathbf{l} + \varphi_J. \tag{18}$$

The path C' goes from L to R - see $Fig.$3. As the result one obtains the relation between V_s and φ

$$2\pi\frac{V_s}{V_R} = -\varphi_J + 2\pi n, \tag{19}$$

where $V_R = \hbar/2m_aR$. The dimensionless energy of the junction (contact) $\bar{W} = W/\pi\hbar n_s V_R$ is given by

$$\bar{W} = \left(\frac{V_s}{V_R}\right)^2 - \frac{\mid j_c^{(m)} \mid}{2\pi n_s V_R}\cos(2\pi\frac{V_s}{V_R} + \varphi_{int}). \tag{20}$$

The minimization of $\bar{W}$ with respect to V_s gives the condition on the ring radius R for the appearance of a spontaneous superflow with $V_s \neq 0$

$$R > R_c = \frac{\hbar}{\pi 2 m_a V_J}. \tag{21}$$

Here, $V_J = j_c^{(m)}/n_s$ is an effective velocity trough the weak link. In that case the π-contact ($\varphi_{int} = \pi$) is realized, i.e. there is a *spontaneous superfluid flow in the ring*. This also means that there is a *spontaneous breaking of thee time-reversal symmetry* in the system. From Eq.(21) one concludes that for small critical currents (velocity V_J) of the weak link the π-contact is realized for large R. Due to the lack of precise experimental data for parameters V_F, V_J, Δ, E_F one can make some qualitative guesses of R_c only. For instance in $UCFAG$ with ^{40}K, by assuming that $E_F \sim (1-10)\,\mu K$, $(\Delta/E_F) \sim 10^{-2}-10^{-3}$, and that $V_J \sim (10^{-2} - 10^{-4})V_{s,c}$ with $V_{s,c} \approx (\Delta/E_F)V_F$ - the critical depairing velocity, one gets $R_c \sim (10^3 - 10^5)$ μm. Since R_c scales inversely with the atomic mass it is larger for lower atomic mass.

V. DISCUSSION AND CONCLUSIONS

In conclusion, we have shown that in ultracold fermionic atomic gases ($UCFAG$) with the mismatch in energies (chemical potentials) of pairing atoms one can realize π-Josephson junction (contact) in SMS weak links. In that case the weak link M is made from $UCFAG$ with the finite mismatch in hyperfine energies (chemical potentials) of the pairing atoms, $\delta \neq 0$. Our proof was given for the case when the energy mismatch δ is of the order of the fermionic superfluid gap Δ, $\delta \sim \Delta$, in which case M is in the normal state but near to the nonuniform $LOFF$ state. However, by analogy with the physics of SFS junctions one expects that the π-Josephson junction can be realized also for very large mismatch, i.e., $\delta \gg \Delta$. In this case the decay-length of the superfluid pairing amplitude induced in the weak link M is very short, i.e. $Q_M^{-1} \sim (V_F/\delta) \ll \xi_0$. In such a case the problem must be attacked by more sophisticated theoretical methods, for instance by the Eilenberger quasiclassical equations in case when $q^{-1} \gg k_F^{-1}$.

Concerning a possible realization of the π-contact in $UCFAG$ it seems that traps by optical lattices are more promising way than inhomogeneous magnetic traps. The possibility for the realization of π-junctions in $UCFAG$ opens a room for numerous speculations about potential applications. For instance, various contacts of the type SM_1M_2S as well as trilayer systems such as M_1SM_2would allow interesting switching properties of these systems. These systems, if they are realizable, would offer a possibility for a new kind of electronics - *hypertronics*. For instance, the computer memory might be realized in weak links $M_{1,2}$, while the logic processing can be done by manipulating the superfluid state. These are open but very attractive problems in this very promising field. One should admire that the realization and manipulation of Josephson junctions in $UCFAG$ represents a considerable experimental challenge.

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